Section 9.2: 1-49 e.o.o., 51-61 odd

1. $\lim_{x\to 2} \frac{x-2}{x^2-4} = \frac{0}{0}$

$\lim_{x\to 2} \frac{1}{2x} = \frac{1}{2(2)} = \boxed{\frac{1}{4}}$

5. $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1-1}{0^2} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\sin x}{2x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\cos x}{2} = \frac{\cos 0}{2} = \boxed{\frac{1}{2}}$

9. a) $\lim_{x\to 0^-} \frac{\sin 4x}{\sin 2x} = \frac{0}{0}$

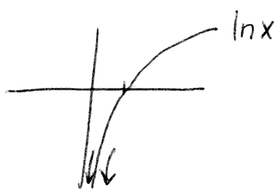
$\lim_{x\to 0^-} \frac{4\cos 4x}{2\cos 2x} = \frac{4(1)}{2(1)} = \frac{4}{2} = \boxed{2}$

b) Also $\boxed{2}$

3. $\lim_{x\to\pi} \frac{\csc x}{1+\cot x} = \lim_{x\to\pi} \frac{\frac{1}{\sin x}}{1+\frac{1}{\tan x}} = \frac{\frac{1}{\sin\pi}}{1+\frac{1}{\tan\pi}} = \frac{\frac{1}{0}}{1+\frac{1}{0}} = \frac{\infty}{1+\infty} = \boxed{\frac{\infty}{\infty}}$

$\lim_{x\to\pi} \frac{\cancel{-}\cancel{\csc x}\cdot\cot x}{\cancel{-}\csc^{\cancel{2}} x} = \lim_{x\to\pi} \frac{\cot x}{\csc x} = \lim_{x\to\pi} \frac{\cos x}{\cancel{\sin x}}\cdot\frac{\cancel{\sin x}}{1} = \lim_{x\to\pi} \cos x = \cos\pi = \boxed{-1}$

17. $\lim_{x\to 0^+} (x\cdot\ln x) = \boxed{0\cdot(-\infty)}$ bc



$\lim_{x\to 0^+} \frac{\ln x}{1/x} = \frac{-\infty}{\infty}$

$\lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} \frac{1}{\cancel{x}}\cdot\frac{-x^{\cancel{2}}}{1} = \lim_{x\to 0^+} -x = -0 = \boxed{0}$

21. $\lim_{x\to 0}(e^x+x)^{1/x} = (e^0+0)^{1/0} = (1+0)^{\infty} = \boxed{1^{\infty}}$

$\ln f(x) = \frac{1}{x}\cdot\ln(e^x+x)$

$\lim_{x\to 0}\frac{\ln(e^x+x)}{x} = \frac{\ln(e^0+0)}{0} = \frac{\ln(1)}{0} = \frac{0}{0}$

$\lim_{x\to 0}\frac{\frac{1}{e^x+x}(e^x+1)}{1} = \lim_{x\to 0}\frac{e^x+1}{e^x+x} = \frac{e^0+1}{e^0+0} = \frac{1+1}{1+0} = \frac{2}{1} = 2$

$\underset{e}{\cancel{\ln}}\lim_{x\to 0} f(x) = \underset{e}{2} \to \lim_{x\to 0} f(x) = \boxed{e^2}$

25. $\lim_{x\to 0^+}\left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{0}\right)^0 = (1+\infty)^0 = \boxed{\infty^0}$

$\ln f(x) = x\cdot\ln(1+\frac{1}{x})$

$\lim_{x\to 0^+}\frac{\ln(1+1/x)}{1/x} = \frac{\ln(1+1/0)}{1/0} = \frac{\ln(\infty)}{\infty} = \frac{\infty}{\infty}$

$\lim_{x\to 0^+}\frac{\frac{1}{1+1/x}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{-\frac{1}{x^2}}} = \lim_{x\to 0^+}\frac{1}{1+1/x} = \frac{1}{1+1/0} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$

$\underset{e}{\cancel{\ln}}\lim_{x\to 0^+} f(x) = \underset{e}{0} \to \lim_{x\to 0^+} f(x) = e^0 = \boxed{1}$

29. $f(\theta) = \frac{\sin 3\theta}{\sin 4\theta}$

θ	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
$f(\theta)$	-0.186	0.7589	0.7501	0.7500	0.7500

Estimate from table : 0.7500

$\lim_{\theta\to 0}\frac{\sin 3\theta}{\sin 4\theta} = \frac{\sin 0}{\sin 0} = \frac{0}{0}$

$\lim_{\theta\to 0}\frac{3\cos 3\theta}{4\cos 4\theta} = \frac{3\cos 0}{4\cos 0} = \frac{3\cdot 1}{4\cdot 1} = \boxed{\frac{3}{4}}$

33. $\lim_{\theta\to 0}\frac{\sin\theta^2}{\theta} = \frac{0}{0}$

$\lim_{\theta\to 0}\frac{\cos(\theta^2)\cdot 2\theta}{1} = \cos 0\cdot 2\cdot 0 = 1\cdot 2\cdot 0 = \boxed{0}$

37. $\lim_{y \to \frac{\pi}{2}} \left(\frac{\pi}{2} - y\right) \tan y = 0 \cdot \tan\frac{\pi}{2} = 0 \cdot \frac{1}{0} = 0 \cdot \infty$

$$\lim_{y \to \frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - y\right)\sin y}{\cos y} = \frac{0 \cdot 1}{0} = \frac{0}{0}$$

$$\lim_{y \to \frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - y\right)\cos y + \sin y(-1)}{-\sin y} = \frac{0 \cdot 0 + 1(-1)}{-1} = \frac{0-1}{-1} = \frac{-1}{-1} = \boxed{1}$$

41. $\lim_{x \to \pm\infty} \frac{3x - 5}{2x^2 - x + 2} = \lim_{x \to \pm\infty} \frac{3\cancel{x}}{2x^{\cancel{2}}} = \lim_{x \to \pm\infty} \frac{3}{2x} = \frac{3}{2(\pm\infty)} = \boxed{0}$

45. $\lim_{x \to 0^+} (1+x)^{1/x} = (1+0)^{1/0} = 1^\infty$

$$\ln f(x) = \frac{1}{x} \cdot \ln(1+x)$$

$$\lim_{x \to 0^+} \frac{\ln(1+x)}{x} = \frac{\ln 1}{0} = \frac{0}{0}$$

$$\lim_{x \to 0^+} \frac{\frac{1}{1+x}}{1} = \lim_{x \to 0^+} \frac{1}{1+x} = \frac{1}{1+0} = \frac{1}{1} = 1$$

$$\cancel{\ln} \lim_{x \to 0^+} f(x) = 1_{e} \rightarrow \lim_{x \to 0^+} f(x) = e^1 = \boxed{e}$$

49. $\lim_{x \to 1} \frac{x^3 - 1}{4x^3 - x - 3} = \frac{1-1}{4-1-3} = \frac{0}{0}$

$$\lim_{x \to 1} \frac{3x^2}{12x^2 - 1} = \frac{3 \cdot 1}{12 \cdot 1 - 1} = \frac{3}{12-1} = \boxed{\frac{3}{11}}$$

51. $\lim_{x \to 1} \frac{\int_1^x \cos t \, dt}{x^2 - 1} = \frac{0}{0}$

$$\lim_{x \to 1} \frac{\cos x}{2x} = \frac{\cos 1}{2 \cdot 1} = \frac{\cos 1}{2} = \boxed{\frac{1}{2}\cos(1)}$$

53. a) $\lim\limits_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}} = \frac{\sqrt{9\infty+1}}{\sqrt{\infty+1}} = \frac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \frac{\cancel{\frac{1}{2}}(9x+1)^{-1/2}\cdot 9}{\cancel{\frac{1}{2}}(x+1)^{-1/2}} = \lim\limits_{x\to\infty} \frac{9\sqrt{x+1}}{\sqrt{9x+1}} = \frac{9\sqrt{\infty+1}}{\sqrt{9\infty+1}} = \frac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \frac{\cancel{9}\cdot\cancel{\frac{1}{2}}(x+1)^{-1/2}}{\cancel{\frac{1}{2}}(9x+1)^{-1/2}\cdot\cancel{9}} = \lim\limits_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}} = \frac{\sqrt{9\infty+1}}{\sqrt{\infty+1}} = \frac{\infty}{\infty}$

No matter how many times you apply L'Hopital's rule, the result is $\frac{\infty}{\infty}$.

b) On the graph, as $x\to\infty$ the limit appears to be 3.

c) $\lim\limits_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}} = \lim\limits_{x\to\infty} \frac{\sqrt{9+1/x}}{\sqrt{1+1/x}} = \frac{\sqrt{9+\frac{1}{\infty}}}{\sqrt{1+\frac{1}{\infty}}} = \frac{\sqrt{9+0}}{\sqrt{1+0}} = \frac{\sqrt{9}}{\sqrt{1}} = \frac{3}{1} = \boxed{3}$

55. $\lim\limits_{x\to 0} \frac{9x-3\sin 3x}{5x^3} = \frac{0-0}{0} = \frac{0}{0}$

$\lim\limits_{x\to 0} \frac{9-9\cos 3x}{15x^2} = \frac{9-9(1)}{0} = \frac{0}{0}$

$\lim\limits_{x\to 0} \frac{27\sin 3x}{30x} = \frac{27(0)}{30(0)} = \frac{0}{0}$

$\lim\limits_{x\to 0} \frac{81\cos 3x}{30} = \frac{81\cos 0}{30} = \frac{81(1)}{30} = \frac{81}{30} = \boxed{\frac{27}{10}}$

57. a) $\lim\limits_{k\to\infty} A_0\left(1+\frac{r}{k}\right)^{kt} = A_0\left(1+\frac{r}{\infty}\right)^{\infty t} = A_0(1+0)^{\infty} = A_0\cdot 1^{\infty}$

$\ln f(k) = A_0 kt\cdot\ln\left(1+\frac{r}{k}\right) = \frac{\ln(1+r/k)}{\frac{1}{A_0 kt}}$

$\lim\limits_{k\to\infty} \frac{\ln(1+r/k)}{\frac{1}{A_0 t k}} = \frac{\ln(1+r/\infty)}{\frac{1}{A_0 t\infty}} = \frac{\ln(1+0)}{\frac{1}{\infty}} = \frac{\ln 1}{0} = \frac{0}{0}$

$\lim\limits_{k\to\infty} \frac{\frac{1}{1+r/k}\cdot\frac{-r}{k^2}}{\frac{-1}{A_0 t k^2}} = \lim\limits_{k\to\infty} \frac{r}{1+\frac{r}{k}}\cdot A_0 t = \frac{A_0 tr}{1+\frac{r}{\infty}} = \frac{A_0 tr}{1+0} = \frac{A_0 tr}{1} = A_0 tr$

$\cancel{\ln} \lim\limits_{k\to\infty} f(k) = A_0 tr \to \lim\limits_{k\to\infty} f(k) = e^{A_0 rt} = \underbrace{e^{A_0}}_{\#}\cdot e^{rt} = \boxed{A_0 e^{rt}}$

57. b) If the number of times per year that the money is compounded approaches ∞, then the money approaches being compounded continuously. The more compounding periods there are per year, the more frequently the money will be compounded.

59. $$A(t) = \int_0^t e^{-x}dx = -e^{-x}\Big|_0^t = \frac{-1}{e^x}\Big|_0^t = \frac{-1}{e^t} + \frac{1}{e^0} = \frac{-1}{e^t} + \frac{1}{1} = \frac{-1}{e^t} + 1$$

$$V(t) = \pi\int_0^t (e^{-x})^2 dx = \pi\int_0^t e^{-2x}dx = \pi \cdot e^{-2x} \cdot \frac{-1}{2}\Big|_0^t = \frac{-\pi}{2e^{2x}}\Big|_0^t$$

$$V(t) = \frac{-\pi}{2e^{2t}} + \frac{\pi}{2\cancel{e^0}(1)} = \frac{-\pi}{2e^{2t}} + \frac{\pi}{2}$$

a) $$\lim_{t\to\infty} A(t) = \lim_{t\to\infty} \frac{-1}{e^t} + 1 = \frac{-1}{e^\infty} + 1 = \frac{-1}{\infty} + 1 = 0 + 1 = \boxed{1}$$

b) $$\lim_{t\to\infty} \frac{V(t)}{A(t)} = \lim_{t\to\infty} \frac{\frac{-\pi}{2e^{2t}} + \frac{\pi}{2}}{\frac{-1}{e^t} + 1} = \frac{\frac{-\pi}{2e^{2\infty}} + \frac{\pi}{2}}{\frac{-1}{e^\infty} + 1} = \frac{\frac{-\pi}{\infty} + \frac{\pi}{2}}{\frac{-1}{\infty} + 1} = \frac{0 + \frac{\pi}{2}}{0 + 1} = \boxed{\frac{\pi}{2}}$$

c) $$\lim_{t\to 0^+} \frac{V(t)}{A(t)} = \lim_{t\to 0^+} \frac{\frac{-\pi}{2e^0} + \frac{\pi}{2}}{\frac{-1}{e^0} + 1} = \frac{-\frac{\pi}{2} + \frac{\pi}{2}}{-1+1} = \frac{0}{0}$$

$$\lim_{t\to 0^+} \frac{\cancel{-}\frac{\pi}{\cancel{2}} e^{-2t}\cdot(\cancel{-2})}{\cancel{-}e^{-t}(\cancel{-1})} = \lim_{t\to 0^+} \frac{\pi e^{-2t}}{e^{-t}} = \lim_{t\to 0^+} \frac{\pi e^t}{e^{2t}} = \lim_{t\to 0^+} \frac{\pi \cancel{e^t}}{e^t \cdot \cancel{e^t}} = \frac{\pi}{e^0} = \frac{\pi}{1} = \boxed{\pi}$$

61. $f(x) = \left(1 + \frac{1}{x}\right)^x$

a) $a^x = e^{x\ln a} \rightarrow \left(1+\frac{1}{x}\right)^x = e^{x\ln(1+\frac{1}{x})}$

Domain of ln is positive, so $1 + \frac{1}{x} > 0 \rightarrow \frac{1}{x} > -1$ when $x > 0$ or $x < -1$

Domain of $f(x)$: $\boxed{(-\infty, -1) \cup (0, \infty)}$

61. b) $\lim_{x\to -1^-} \left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{-1}\right)^{-1} = (1-1)^{-1} = 0^{-1} = \frac{1}{0^1} = \frac{1}{0} = \boxed{\infty}$

c) $\lim_{x\to -\infty} \left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{-\infty}\right)^{-\infty} = (1+0)^{-\infty} = 1^{-\infty}$

$\ln f(x) = x \cdot \ln\left(1+\frac{1}{x}\right) = \dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$

$\lim_{x\to -\infty} \dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \dfrac{\ln\left(1+\frac{1}{-\infty}\right)}{\frac{1}{-\infty}} = \dfrac{\ln(1+0)}{0} = \dfrac{\ln 1}{0} = \dfrac{0}{0}$

$\lim_{x\to -\infty} \dfrac{\frac{1}{1+1/x} \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \lim_{x\to -\infty} \dfrac{1}{1+\frac{1}{x}} = \dfrac{1}{1+\frac{1}{-\infty}} = \dfrac{1}{1+0} = \dfrac{1}{1} = 1$

$\cancel{e^{\ln}} \lim_{x\to -\infty} f(x) = e^1 \rightarrow \lim_{x\to -\infty} f(x) = e^1 = \boxed{e}$