Binomial Theorem (Section 9.2)

* Powers of Binomials: $(a+b)^0 = 1$

$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$\vdots$$

* You can use the calculator to get the coefficients!

ex: $(a+b)^6$ In the Calc type:

$\boxed{6}\ \boxed{{}_nC_r}\ \boxed{\{}\ 0,1,2,3,4,5,6\ \boxed{\}} = \underbrace{1, 6, 15, 20, 15, 6, 1}_{\text{What does this mean?}}$

Math/Prob/#3 2nd/(2/)

$$= 1a^6b^0 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6a^1b^5 + 1a^0b^6$$
$$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

* All exponents add up to six!

* Pascal's Triangle:

```
            1
           1 1
          1 2 1
         1 3 3 1
        1 4 6 4 1
      1 5 10 10 5 1
    1 6 15 20 15 6 1
```

Same as what we got using the calc!

* To get from one line to another:

```
  1  3  3  1
1  4  6  4  1
```

Always start w/ a one on the outside, then add the two numbers above to get the new coefficient. And... end w/ a one!

* To get just ONE term w/ it's coefficient: use ${}_nC_r$.

ex: Find the coefficient of x^{10} in the expansion of $(x+2)^{15}$.

a b

$$= {}_{15}C_{10}\, x^{10} (2)^5$$
$$= 3003\, x^{10} \cdot 32$$
$$\boxed{= 96{,}096\, x^{10}}$$

ex: Expand: $(2x-y^2)^4 \rightarrow 1\ 4\ 6\ 4\ 1$

$a=2x \quad b=-y^2$

substitute a & b in

$= a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$= (2x)^4+4(2x)^3(-y^2)+6(2x)^2(-y^2)^2+4(2x)(-y^2)^3+(-y^2)^4$

$\boxed{= 16x^4-32x^3y^2+24x^2y^4-8xy^6+y^8}$

ex: Expand: $(x-3y)^5 \rightarrow 1\ 5\ 10\ 10\ 5\ 1$

$a=x \quad b=-3y$

substitute in a & b

$= a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

$= (x)^5+5(x)^4(-3y)+10(x)^3(-3y)^2+10(x)^2(-3y)^3+5x(-3y)^4+(-3y)^5$

$\boxed{= x^5-15x^4y+90x^3y^2-270x^2y^3+405xy^4-1215y^5}$

ex: #23 $(\sqrt{x}-\sqrt{y})^6 \rightarrow 1\ 6\ 15\ 20\ 15\ 6\ 1$

$a=\sqrt{x}$

$b=\sqrt{y}$

$= a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$

$= (\sqrt{x})^6+6(\sqrt{x})^5(\sqrt{y})+15(\sqrt{x})^4(\sqrt{y})^2+20(\sqrt{x})^3(\sqrt{y})^3+15(\sqrt{x})^2(\sqrt{y})^4+6(\sqrt{x})(\sqrt{y})^5$
$+(\sqrt{y})^6$

$\boxed{= x^3+6x^{5/2}y^{1/2}+15x^2y+20x^{3/2}y^{3/2}+15xy^2+6x^{1/2}y^{5/2}+y^3}$

ex: Find the x^5y^8 term of $(x+y)^{13}$. $\rightarrow {}_{13}C_5\,(x)^5(y)^8 = \boxed{1287x^5y^8}$

ex: Find the 4^{th} term in the expansion of $(3x+2y)^9$.

${}_9C_4\,(3x)^4(2y)^5 = 126(81x^4)(32y^5)$

$\boxed{= 326{,}592x^4y^5}$