Probability: Section 9.3

* Sample Space: all possible outcomes of an event.

ex: Flipping a coin: H/T → P(H) = 1/2, P(T) = 1/2

Rolling a number cube: 1/2/3/4/5/6 → P(1) = 1/6

Rolling two number cubes: 2/3/4/5/6/7/8/9/10/11/12

* 36 outcomes

rolling a 7: 1,6 / 2,5 / 3,4 / 4,3 / 5,2 / 6,1

P(7) = 6/36 = 1/6

Picking a card from a standard deck of cards:

52 cards

4 suits: Hearts, diamonds, spades, clovers → P(Heart) = 13/52 = 1/4

2 colors: Red, black → P(Black) = 26/52 = 1/2

3 face cards per suit: Jack, King, Queen → P(Queen) = 4/52 = 1/13

0 Jokers, 13 cards each suit

* Probability: the likelihood of an event, $0 \le P(E) \le 1$

$$P(E) = \frac{\#\text{ of successes}}{\text{TOTAL \# of outcomes}}$$

0: never happening; 1: certain to happen

ex: P(Red Jack) = P(Heart/Diamond) = $\frac{2}{52} = \boxed{\frac{1}{26}}$

ex: P(rolling a 4) = $\boxed{\frac{3}{52}}$

1/3, 2/2, 3/1

* Independent Events -vs- Dependent Events:

Independent: one event does not effect the second

Dependent: one event determines/changes the likelihood of the second.

ex: Rolling a number cube multiple times. I/D? independent

ex: Picking two cards out of a standard deck. I/D dependent: changes the sample space

* Probability of independent events: you can multiply the probabilities of each event.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

* Probability of dependent events: called "conditional probability."

$$P(A|B) = \frac{P(A \text{ and } B)}{P(A)}$$

"probability of A GIVEN B"

ex: If A and B are independent events:

a) find: $P(A \text{ and } B)$ if $P(A) = \frac{2}{3}$, $P(B) = \frac{7}{10}$

$P(A) \cdot P(B) = P(A \text{ and } B)$

$\frac{2}{3} \cdot \frac{7}{10} = \frac{14}{30} = \boxed{\frac{7}{15}}$

b) find $P(A)$ if $P(A \text{ and } B) = \frac{9}{20}$, $P(B) = \frac{3}{5}$

$\frac{5}{3}\left(P(A) \cdot \frac{3}{5} = \frac{9}{20}\right)\frac{5}{3}$

$P(A) = \frac{45}{60} = \boxed{\frac{3}{4}}$

ex: If A and B are dependent events:

a) find $P(A|B)$ if $P(A \text{ and } B) = \frac{5}{12}$, $P(A) = \frac{1}{2}$

$P(A|B) = \frac{P(A \text{ and } B)}{P(A)} = \frac{5/12}{1/2} = \frac{5}{12} \cdot \frac{2}{1} = \frac{10}{12} = \boxed{\frac{5}{6}}$

b) find $P(A)$ if $P(A \text{ and } B) = \frac{1}{2}$, $P(A|B) = \frac{2}{3}$

$\frac{2}{3} = \frac{1/2}{P(A)}$

$P(A) = \frac{1/2}{2/3} = \frac{1}{2} \cdot \frac{3}{2} = \boxed{\frac{3}{4}}$

c) find $P(B)$ if $P(A \text{ and } B) = \frac{4}{5}$, $P(B|A) = \frac{9}{11}$

$P(B|A) = \frac{P(A \text{ and } B)}{P(B)} = \frac{4/5}{P(B)}$

$\frac{9}{11} = \frac{4/5}{P(B)}$

$P(B) = \frac{4/5}{9/11} = \frac{4}{5} \cdot \frac{11}{9} = \boxed{\frac{44}{45}}$

* Probability using Combinations:

$$P(\text{event}) = \frac{\text{Combinations for success}}{\text{Combinations TOTAL}}$$

ex: In a class of 15 boys and 17 girls a group of 5 is chosen. What is the probability that all 5 students are girls?

32 total

$$P(\text{all girls}) = \frac{{}_{17}C_{5}}{{}_{32}C_{5}} = \frac{6188}{201376} = \boxed{0.0307}$$

(17: girls; 5: # in group; 32: total)

ex: 4 Red, 2 Blue, 7 green marbles. A group of 4 is chosen, what is the probability all are red?

13 total
4 Red

$$P(\text{all red}) = \frac{{}_{4}C_{4}}{{}_{13}C_{4}} = \frac{1}{715} = \boxed{0.0014}$$

(4: red; 13: total; 4: # in group)

What is the probability that 2 are red?

13 total

$$P(2 \text{ red}) = \frac{{}_{4}C_{2} \cdot {}_{9}C_{2}}{{}_{13}C_{4}} = \frac{6 \cdot 36}{715} = \frac{216}{715} = \boxed{.302}$$

(4: red; 9: not red; 13: total; these two should equal the total.)